**9) REAL LIFE PROBLEMS IN CONICS**

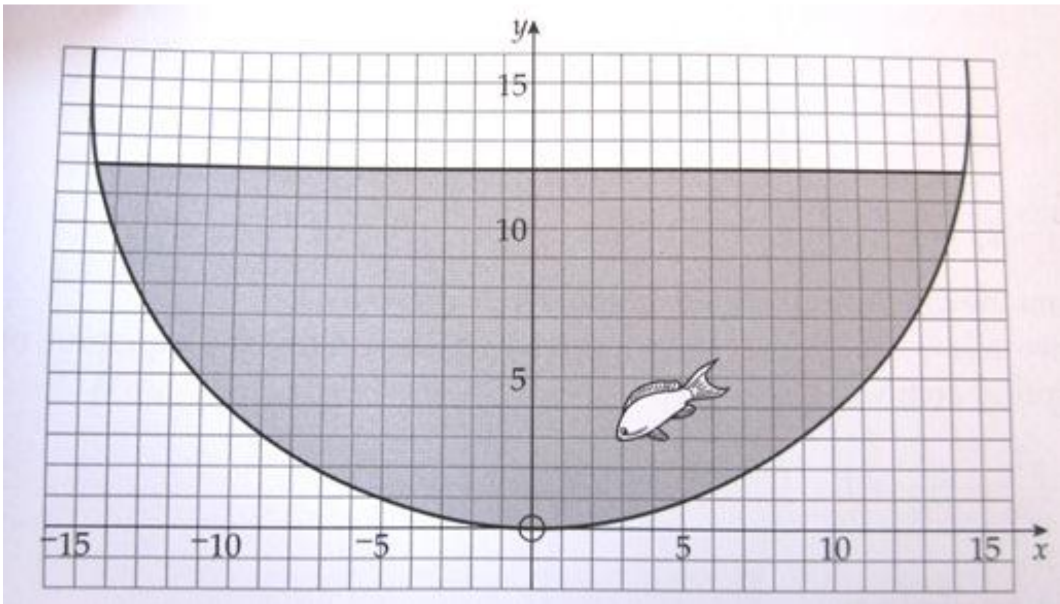
Real life problems in Conics may involve any of the following:

- Modelling in contexts like bridges, tunnels, cross-sections of 3D shapes
- Equations of tangents or normals
- Points of intersections between lines and conics

Examples:

1) A fish bowl is in the shape of a hemisphere, and therefore has a semi-circular cross-section. The bowl has a diameter of 30cm and is filled with water to a depth of 11cm. What is the surface area of the top of the water?

The surface area at the top of the water is in the shape of a circle, so we will need the radius of the water when the depth of the water is 11cm. We can form an equation for a circle, where the bottom part of the circle models the cross-section of the fish bowl. If we let the bottom of the circle rest on the origin, then all we need to do is substitute $y = 11$ into the equation to find $x$, which would be our radius.




The radius of our circle is 15cm. The circle has been translated up by 15, so the equation of the circle is: $x^2 + (y - 15)^2 = 15^2$. Substitute $y = 11$ into the equation:

$x^2 + (11 - 15)^2 = 225$

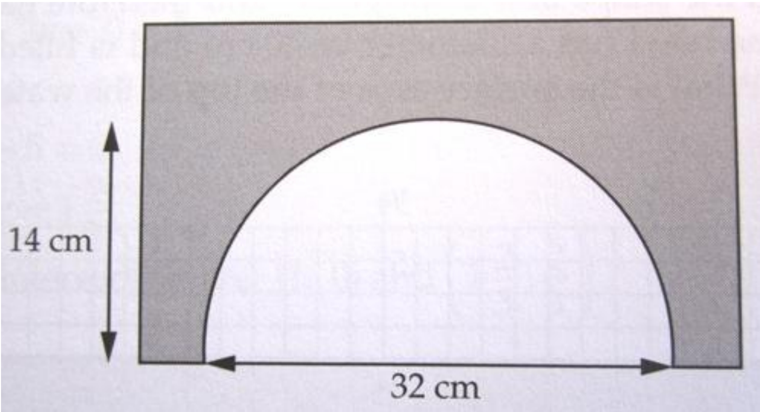
$x^2 = 209$

$x = \pm 14.46$ so the radius is 14.46 cm when the water is at a depth of 11cm.

The surface area of the water is therefore:

$A = \pi \cdot (14.46)^2 = 656.6 \text{ cm}^2$

2) The diagram below shows the cross-section of a bridge for a toy train. The highest point of the arch is 14 cm, and the span at the base of the arch is 32 cm. Find the equation of the elliptical arch of the bridge.




We can use the shape of an ellipse to model the bridge. From the diagram, $a = 16, b = 14$.

If we make the centre of the ellipse the origin, then our equation is: $\frac{x^2}{16^2} + \frac{y^2}{14^2} = 1$.

Worksheet – Ex 3.5C M3, and Merit Revision